次の極限値を求めよ。（Ｎｏ．１）

(1) $\lim_{n\to\infty}\frac{1}{n}\left(\sin\frac{\pi}{2n}+\sin\frac{2\pi}{2n}+\sin\frac{3\pi}{2n}+\cdots+\sin\frac{n\pi}{2n}\right)$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\sin\frac{k\pi}{2n}$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\sin\left(\frac{\pi}{2}\cdot\frac{k}{n}\right)$

$=\int_0^1\sin\frac{\pi}{2}x\,dx=\left[-\frac{2}{\pi}\cos\frac{\pi}{2}x\right]_0^1=-\frac{2}{\pi}\left(\cos\frac{\pi}{2}-\cos 0\right)=\frac{2}{\pi}$

(2) $\lim_{n\to\infty}\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{n+1}}+\frac{1}{\sqrt{n+2}}+\frac{1}{\sqrt{n+3}}+\cdots+\frac{1}{\sqrt{2n}}\right)$

$=\lim_{n\to\infty}\frac{1}{n}\left(\frac{\sqrt{n}}{\sqrt{n+1}}+\frac{\sqrt{n}}{\sqrt{n+2}}+\frac{\sqrt{n}}{\sqrt{n+3}}+\cdots+\frac{\sqrt{n}}{\sqrt{2n}}\right)$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{\sqrt{n}}{\sqrt{n+k}}$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{\sqrt{1+\frac{k}{n}}}$

$=\int_0^1\frac{1}{\sqrt{1+x}}dx=\left[2(1+x)^{\frac{1}{2}}\right]_0^1=2\left(2^{\frac{1}{2}}-1\right)=2(\sqrt{2}-1)$

(3) $\lim_{n\to\infty}\frac{\pi}{n}\left(\sin^3\frac{\pi}{n}+\sin^3\frac{2\pi}{n}+\sin^3\frac{3\pi}{n}+\cdots+\sin^3\frac{n\pi}{n}\right)$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\pi\sin^3\frac{k}{n}\pi$

$=\int_0^1\pi\sin^3\pi x\,dx$

$=\pi\int_0^1(1-\cos^2\pi x)\sin\pi x\,dx=\pi\int_1^{-1}(1-t^2)(-\pi)dt=\pi^2\int_{-1}^{1}(1-t^2)dt=2\pi^2\int_0^1(1-t^2)dt=2\pi^2\left[t-\frac{1}{3}t^3\right]_0^1=\frac{4}{3}\pi^2$

(4) $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\sin^2\frac{\pi k}{4n}$

$=\int_0^1\sin^2\frac{\pi}{4}x\,dx=\int_0^1\frac{1-\cos 2\frac{\pi}{4}x}{2}dx=\frac{1}{2}\int_0^1\left(1-\cos\frac{\pi}{2}x\right)dx=\frac{1}{2}\left[x-\frac{2}{\pi}\sin\frac{\pi}{2}x\right]_0^1=\frac{1}{2}\left(1-\frac{2}{\pi}\right)=\frac{\pi-2}{2\pi}$

(5) $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{k}{\sqrt{n^2+k^2}}$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{\frac{k}{n}}{\sqrt{1+\left(\frac{k}{n}\right)^2}}$

$=\int_0^1\frac{x}{\sqrt{1+x^2}}dx=\int_1^2\frac{1}{\sqrt{t}}\frac{1}{2}dt=\frac{1}{2}\left[2t^{\frac{1}{2}}\right]_1^2=\sqrt{2}-1$

(6) $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n^2+k^2}$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{\frac{k}{n}}{1+\left(\frac{k}{n}\right)^2}$

$=\int_0^1\frac{x}{1+x^2}dx=\int_0^1\frac{1}{1+t}\frac{1}{2}dt=\frac{1}{2}[\log|1+t|]_0^1=\frac{1}{2}\log 2$

(7) $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n}{(n+k)(3n+k)}$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{\infty}\frac{1}{\left(1+\frac{k}{n}\right)\left(3+\frac{k}{n}\right)}$

$=\int_0^1\frac{1}{(1+x)(3+x)}dx$

$=\frac{1}{2}\int_0^1\left(\frac{1}{1+x}-\frac{1}{3+x}\right)dx=\frac{1}{2}[\log|1+x|-\log|3+x|]_0^1=\frac{1}{2}(\log 2-\log 4+\log 3)=\frac{1}{2}\log\frac{3}{2}$

(8) $\lim_{n\to\infty}\frac{1}{n}(\sqrt[n]{e}+\sqrt[n]{e^2}+\sqrt[n]{e^3}+\cdots+\sqrt[n]{e^n})$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\sqrt[n]{e^k}=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}e^{\frac{k}{n}}$

$=\int_0^1 e^x dx=[e^x]_0^1=e-1$

(9) $\lim_{n\to\infty}\left(\frac{a}{n+a}+\frac{a}{n+2a}+\frac{a}{n+3a}+\cdots+\frac{a}{n+na}\right)$　（ただし、aは正の定数）

$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{a}{n+ka}$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{a}{1+a\frac{k}{n}}$

$=\int_0^1\frac{a}{1+ax}dx=\int_0^a\frac{1}{1+t}dt=[\log|1+t|]_0^a=\log(1+a)$